3.19. Features of Validity

We've seen numerous arguments illustrating the nice parallel between validity, on the one hand, and intuitive "following from," or "entailing," on the other. But occasionally this parallel is strained – yielding arguments which do technically qualify as valid, but where we are uncomfortable saying that the conclusion "follows from" the premises.

And for all their variety, each of these peculiar cases stems from the nature of a **validity counterexample** for an argument: a valuation where (i) the premises of the argument are all true, but (ii) the conclusion is false. In what follows we survey several different and very unintuitive ways in which an argument can evade one or the other of these two conditions – thereby qualifying as a valid argument.

1. Unintuitively Valid Arguments: Four Kinds. Two such cases are familiar from informal logic.

***First*, any sentence follows validly from itself**. The following argument, for instance, is perfectly valid.

1. P

―――――

∴ P

And we needn't bother with truth tables to see that this is so: in any possible situation (valuation) where "P" is true, "P" is true. It's thus impossible to have a validity counterexample for this argument – for any valuation clearing the first hurdle (making the premises true) fails on the second (making the conclusion false).

Of course this argument will never *convince* anyone of P – for reasons rehearsed already in our discussion of pragmatics.[1] Briefly: anyone in need of convincing doesn't already believe P. But not believing P, they will not judge the argument to pass the true premises requirement – and hence will not find the argument convincing.

[1] In Chapter 2, section X.

***Second*, adding premises to a valid argument always yields a valid argument**. The following familiar argument, for instance, is at this point notoriously valid.

1. $(P \vee Q)$
2. $\sim P$

$\therefore Q$

And adding any premise whatsoever – however irrelevant – yields a larger valid argument.

1. $(P \vee Q)$
2. $\sim P$
3. X

$\therefore Q$

Truth tables bear this out: in the one valuation making all three premises true, the conclusion is true.

			(3)	(1)	(2)	∴
	P	**Q**	**X**	**(P ∨ Q)**	**~P**	**Q**
	1	1	1	1	0	1
	1	1	0	1	0	1
	1	0	1	0	0	0
	1	0	0	0	0	0
⇨	0	1	**1**	**1**	**1**	**1**
	0	1	0	**1**	**1**	1
	0	0	1	0	1	0
	0	0	0	0	1	0

Note that the first two premises, "(P ∨ Q)" and "~P," are true together in two valuations: the fifth (emphasized here) and the sixth. If adding an additional premise has any effect, it can only be (as in this case) to *reduce* the number of valuations making all the premises true. (Adding the third premise, "X," weeds out Valuation 6 as one making *all* the premises true.) By making it that much harder to have *all the premises true*, (the first requirement for a validity counterexample), we can only *lower* the chances of having of a validity counterexample. So if there were no counterexamples to begin with (because the original argument was valid), adding further premises cannot introduce a counterexample. The more premises we heap on, the more immune the argument becomes to counterexamples.

The next two cases push this strategy – making it harder for any situation to qualify as a counterexample – to the limit in different ways. These are the third and fourth examples of unintuitive validity.

Third: **a tautology follows validly from any premises**. So the tautology "(~P ∨ P)" follows validly from, e.g., "X".

1. X

∴ (P ∨ ~P)

	(1)		**∴**
P	**X**	~P	**(P ∨ ~P)**
1	**1**	0	**1**
1	0	0	1
0	**1**	1	**1**
0	0	1	1

Since a tautology is true in every valuation, an argument with a tautology as its conclusion is one whose conclusion is never false in any valuation. That means no valuation can meet the second requirement for being a counterexample: making the conclusion false. An argument with a tautology as conclusion is immune from any counterexample – regardless of when its premise(s) are true, and when they're not.

Fourth, **any conclusion follows validly from inconsistent premises**. In the simplest case: an argument with a contradiction as premise is bound to be valid.

So the argument "(P ∧ ~P) ∴ X" is valid.

1. (P ∧ ~P)

∴ X

			(1)	**∴**
P	X	~P	**(P ∧ ~P)**	**X**
1	1	0	0	1
1	0	0	0	0
0	1	1	0	1
0	0	1	0	0

There being no valuations which make the premises true, there are certainly none making (i) the premises true and (ii) the conclusion false. So no counterexample is possible.

Indeed, since counterexamples are blocked by the premise alone, it doesn't matter which sentence acts as conclusion. Hence **any and every sentence follows validly from a contradiction**. That makes a contradiction especially poisonous, in terms of entailment: assuming we do not believe that every sentence is true – and so don't wish to be committed to every sentence – we must resist believing a contradiction.

More generally: **any inconsistent set of premises will** likewise **validly entail any and every sentence**.

1. P
2. ~P

∴ X

(1)		**(2)**	**∴**
P	X	~P	**X**
1	1	0	**1**
1	0	0	0
0	1	1	**1**
0	0	1	0

The set of sentences {P, ~P} is unsatisfiable (inconsistent), since no valuation makes both sentences true. And once again, this means no validity counterexample is possible.

2. Validity, Relevance, and Pragmatics. Opinions differ over how we should address these oddities these cases present. One strategy promoted in **relevance logic** is to add the further requirement that the premises and conclusion must share at least one sentence letter.[2] That would effectively block "X" from entailing the tautology "$(P \vee {\sim}P)$," and the contradiction "$(P \wedge {\sim}P)$" from entailing "X".

That move would not, however, prevent the first two cases: a sentence entailing itself, and an argument with extra, unnecessary premises qualifying as valid.

As noted in the earlier discussion of pragmatics, a different option here is to leave conditions on validity as they stand, and to explain the unintuitiveness of these cases instead as violation of some conditions on communication, above and beyond matters of validity.[3] On that line of argument, our intuitions about which arguments are acceptable are guided by a number of factors. One of these factors is validity; but another is the pragmatic conditions on communication. In that case the above arguments are indeed bad, but not *logically* bad. They are instead pragmatically bad.

That pragmatic approach to the peculiarities of validity has the happy side-effect of sparing us the need to tinker with conditions for validity, since the admitted oddness of these sorts of arguments is no longer taken to be a malfunction in our standards of validity. Admittedly, that move somewhat complicates the easy parallel previously suggested between validity and intuitive "following from"; for we could then say at best that validity and intuitive following-from run parallel *so long as no pragmatic conditions are violated*. But that mild revision might be a reasonable price to pay if it permits us to continue using the traditional standards of validity in formal logic.

[2] As in, e.g., (Anderson and Belnap1975).
[3] Most famously, in Grice (196x).